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Class: XII th D
Subject: Mathematics
Date: 13/09/2024 13/9/24.
Invigilator's Sign.
Session: 2024-25

Roll No. | 2 | 2 |

SET – A
SUBJECT CODE-041

Time allowed: 3 Hours **Maximum Marks: 80**

- Please check that this question paper contains 5 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer book by the candidate.
- Please check that this question paper contains 38 questions.
- Please write down the serial number of the question in the answer book before attempting it.
- 15 minutes time has been allotted to read this question paper. The candidates will read the question paper only and will not write any answer on the answer book during this period.

General Instructions:
Read the following instructions very carefully and follow them:

i. *This question paper contains 38 questions. **All** questions are compulsory.*
ii. *Question paper divided into **FIVE** Sections – Section **A, B, C, D** and **E.***
iii. *In **Section A** – Question Number 1 to 18 are Multiple Choice Questions **(MCQ)** type and Question Number 19 & 20 are Assertion-Reason based questions of 1 mark each.*
iv. *In **Section B** - Question Number 21 to 25 are Very Short Answer **(VSA)** type questions of 2 marks each.*
v. *In **Section C** - Question Number 26 to 31 are Short Answer **(SA)** type questions carrying 3 marks each.*
vi. *In **Section D** - Question Number 32 to 35 are Long Answer **(LA)** type questions carrying 5 marks each.*
vii. *In **Section E** - Question Number 36 to 38 are case study based questions carrying 4 marks each where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.*
viii. *There is no overall choice. However, an internal choice has been provided in 2 questions in Section-B, 3 questions in Section-C, 2 questions in Section-D and 2 questions in Section-E.*
ix. *Use of calculators is **NOT** allowed.*

SECTION - A
(Multiple Choice Questions)
Each question carries 1 mark.

Select the correct options out of the four given options:

1. Let A= {1, 2, 3}. Then the number of equivalence relations containing (1,3) is
(a) 1 (b) 2 (c) 4 (d) 5

2. If the set A contains 5 elements and set B contains 6 elements, then the number of one-one mapping from A to B is
(a) 720 (b) 120 (c) 0 (d)144

3. The value of $\tan^{-1}(-1) + \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

4. If $A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$, $B=\begin{bmatrix}x & 0\\ 1 & 1\end{bmatrix}$ and $A= B^2$, then x equal to

(a) -1 (b) 1 (c) -2 (d) 2

5. If matrix $A=\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$ and $A^2 = xA$, then the value of x is equal to

(a) 2 (b) 4 (c) 6 (d) 8

6. If A is a 3×3 matrix, such that $\text{adj}(4A) = \lambda(\text{adj } A)$, then the value of λ is

(a) 16 (b) 20 (c) 24 (d) 32

7. If A and B are square matrices of order 3 such that $|A| = 5, |B| = 3$, then the value of $|3AB|$ is

(a) 45 (b) 135 (c) 225 (d) 405

8. If A is an invertible matrix of order 3×3 such that $|A| = 3$, then $|A(\text{Adj } A)|$ is

(a) 9 (b) 18 (c) 27 (d) 81

9. The area of a triangle with vertices (-3, 0), (3, 0) and (0, k) is 9 sq. units. Then the value of k is

(a) -9 (b) 3 (c) 6 (d) 9

10. If $f(x) = \begin{cases}\frac{1-\cos 2x}{x^2}, & x \neq 0\\ k, & x = 0\end{cases}$ is continuous at $x = 0$, then the value of k is equal to

(a) -2 (b) -1 (c) 1 (d) 2

11. Differential coefficient of $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\sin^{-1} x^2$ is

(a) $\frac{-2x}{\sqrt{1+x^2}}$ (b) $\frac{2x}{\sqrt{1+x^2}}$ (c) $\frac{-\sqrt{1+x^2}}{2x}$ (d) $\frac{\sqrt{1+x^2}}{2x}$

12. If $x^{2/3} + y^{2/3} = a^{2/3}$, then $\frac{dy}{dx}$ is equal to

(a) $-\sqrt{\frac{x}{y}}$ (b) $\sqrt{\frac{x}{y}}$ (c) $\sqrt[3]{\frac{x}{y}}$ (d) $-\sqrt[3]{\frac{y}{x}}$

13. The total revenue in rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. Then the marginal revenue, when $x = 15$ is

(a) 116 (b) 96 (c) 90 (d) 126

14. It is given that at $x = 1$, the function $x^4 - 62x^2 + px + 9$ attains its maximum value, on the interval [0,2].Then the value of p is

(a) 120 (b) 124 (c) 128 (d) 130

15. $\int \frac{x}{(x-1)(x-2)}\,dx$ equals

(a) $\log\left|\frac{(x-1)^2}{x-2}\right| + C$ (b) $\log\left|\frac{(x-2)^2}{x-1}\right| + C$ (c) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right| + C$ (d) $\log|(x-1)(x-2)| + C$

16. $\int e^x \frac{(x-3)}{(x-1)^3}\,dx$ equals (a) $\frac{e^x}{x-1} + C$ (b) $\frac{e^x}{(x-1)^2} + C$ (c) $-\frac{e^x}{x-1} + C$ (d) $-\frac{e^x}{(x-1)^2} + C$

17. If the minimum value of an objective function $Z = px + qy$, where p, q occurs at two points (3,0) and (1,1), then
(a) $p = 2q$ (b) $p = q$ (c) $p = 3q$ (d) $q = 2p$

18. The number of solutions of the system of inequalities:
$x + 2y \leq 3,\ 3x + 4y \geq 12,\ x \geq 0,\ y \geq 1$ is
(a) 0 (b) 2 (c) finite (d) infinite

Assertion- Reason Based Questions

In the following questions **19** and **20,** a statement of Assertion **(A)** is followed by a statement of Reason **(R).** Choose the correct answer out of the following choices:
(a) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A).**
(b) Both **(A)** and **(R)** are true and **(R)** is not the correct explanation of **(A).**
(c) **(A)** is true but **(R)** is false.
(d) **(A)** is false but **(R)** is true.

19. **Assertion (A):** $\cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \frac{13\pi}{6}$
Reason (R): $\cos^{-1}(\cos\theta) = \theta$ when $\theta \in [0, \pi]$

20. **Assertion (A):** $\int_3^6 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}dx = \frac{3}{2}$
Reason(R): $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx.$

SECTION - B

This section comprises of Very Short Answer (VSA) type questions of 2 marks each.

21. Consider the function f: **N** → **N** given by $f(x) = x^2 + x + 1$. Find whether it is bijective or not.

22. Draw the graph of $\cot^{-1} x$. Also, write its domain and range.

23. If A is a square matrix such that $A^2 = I$, then find the value of $(A + I)^3 + (A - I)^3 - 7A$.

24. Differentiate $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right), -\pi < x < \pi$ with respect to x.

Or

Find $\frac{dy}{dx}$, if $y = \sqrt{\log\left\{\sin\left(\frac{x^3}{5} - 7\right)\right\}}$.

25. Evaluate: $\int \frac{\sin(x-a)}{\sin(x+a)}\,dx$ **Or** Evaluate: $\int \frac{1}{\sqrt{16-6x-x^2}}\,dx$

SECTION – C

This section comprises of Short Answer (SA) type questions carrying 3 marks each.

26. If the matrix $A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew-symmetric, find the values of a, b and c.

27. If $y = e^{m\sin^{-1}x}$, prove that: $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y$.

Or

If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

28. Find the intervals in which the function $f(x) = \sin x + \cos x,\ [0, 2\pi]$ is (a) increasing (b) decreasing.

29. Evaluate: $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$. **Or** Evaluate: $\int \frac{x}{(x-1)^2(x+2)}\,dx$.

30. Evaluate: $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}dx$. **Or** Evaluate: $\int \frac{5x-2}{1+2x+3x^2}\,dx$

31. If $y = (\log x)^{\cos x} + \frac{x^2+1}{x^2-1}$, then find $\frac{dy}{dx}$.

SECTION – D

This section comprises of Long Answer (LA) type questions carrying 5 marks each.

32. (a) If $y\sqrt{1+x^2} = \log\left(\sqrt{x^2+1}-x\right)$, prove that: $(x^2+1)\frac{dy}{dx} + xy + 1 = 0$.

(b) If $x = a(1+\cos\theta), y = a(\theta+\sin\theta)$, then find $\frac{d^2y}{dx^2}$ at $\theta = \pi/2$.

Or

(a) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$, prove that $\frac{dy}{dx} = \frac{-1}{(x+1)^2}$.

(b) If $(a+bx)e^{y/x} = x$, prove that: $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

33. Minimize and maximize (Using graphical method): $Z = 5x + 2y$

subject to constraints: $x - 2y \leq 2, 3x + 2y \leq 12, -3x + 2y \leq 3, \quad x, y \geqslant 0$.

34. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

35. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations:

$x + 2y + z = 4, \; -x + y + z = 0, \; x - 3y + z = 2.$

Or

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 2 & -2 \end{bmatrix}$, find A^{-1} and hence solve the system of equations: $3x + 2y - 2z = 3$, $2x - y + z = 2, x + 2y + 3z = 6$.

SECTION – E

This section comprises of 3 case study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two sub-parts (I) and (II) of marks 2 each.

36. An inspection was conducted in a School at Delhi. The inspection team visited class XII and selected two sets A, B of three students each. $A = \{b_1, b_2, b_3\}$ and $B = \{g_1, g_2, g_3\}$, where b_i, g_i represent particular boy, girl respectively, $i = 1,2,3$.

Based on the above information, answer the following questions:

(I) How many relations are possible from A to B?

(II) How many functions can be defined from A to B?

(III) Find the number of reflexive relations on set A.

Or

(III) Find the number of equivalence relations on set B.

37. For any function f(x)

$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, where $a < c < b$.

Based on the above information, answer the following questions:

(I) Evaluate: $\int_{-1}^{2} f(x)\,dx$, where $f(x) = \begin{cases} 2x+3, & \text{if } x \le 1 \\ x-4, & \text{if } x > 1 \end{cases}$

(II) Evaluate: $\int_{-1}^{1} e^{|x|}dx$.

(III) Evaluate: $\int_0^2 |x-1|\,dx$.

Or

(III) Evaluate: $\int_{-3}^{3} |x+2|\,dx$.

38. Read the case study problem and answer the following questions:

In a mathematics lab a student observe that water is running into a conical vessel,15 cm deep and 5cm in radius, at the rate of 0.1 cm^3/sec.When the water is 6 cm deep, find at what rate is

(I) the water level rising?

(II) the water-surface area increasing?