MID TERM EXAMINATION (2024-25)

CLASS - XII

SUBJECT - MATHEMATICS

Time : 3 Hours **MM : 80**

General Instructions:

(i) All 38 questions in this questions paper are compulsory.

(ii) Question paper a divided into 5 sections A, B, C, D and E.

(iii) In section A - Q. 1-18 are MCQs and Q. 19 - 20 are Assertion-Reason based and contain 1 mark each.

(iv) Section B - Q. 21 - 25 carry 2 marks each.

(v) Section C - Q. 26 - 31 carry 3 marks each.

(vi) Section D – Q. 32 - 35 carry 5 marks each.

(vii) Section E - Q. 36 - 38 are case study based Questions & carry 4 marks each.

SECTION-A

1. The function $f(x) = [x]$ is

   (a) One-one onto (b) Many-one onto

   (c) One-one into (d) Many-one into

2. If R is a relation in R as aRb if $a \geq b$, then R is

   (a) An equivalence relation

   (b) Reflexive, transitive but not symmetric

   (c) Neither transitive nor reflexive but symmetric

   (d) Symmetric transitive but not reflexive

3. If $\cos^{-1}(x^2-7x+12) = (2n+1)\frac{\pi}{2}$, $\forall n \in z$ then x =

   (a) -2 (b) 4

   (c) -3 (d) 5

4. $\cos\left[\frac{\pi}{3}+\cos^{-1}\left(\frac{-1}{2}\right)\right]$ is equal to

   (a) 0 (b) $\frac{1}{2}$

   (c) -1 (d) 1

5. The Matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$ is a

(a) Diagonal matrix
(b) scalar matrix
(c) Identity matrix
(d) skew symmetric matrix

6. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $KA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a and b respectively are

(a) −6, −12, −18
(b) −6, −4, −9
(c) −6, 4, 9
(d) −6,12,18

7. If $A = \begin{bmatrix} x & 2 \\ 4 & 3 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{1}{6} & \frac{-1}{12} \\ \frac{-1}{6} & \frac{4}{9} \end{bmatrix}$ then value of x is

(a) 8
(b) $\frac{28}{3}$
(c) $\frac{32}{3}$
(d) 10

8. The area of triangle with vertices (–3,0), (3,0) and (0,k) is 6 sq. units. The value of k is

(a) 6
(b) 3
(c) 2
(d) 4

9. If $A = \begin{bmatrix} 2 & k & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$ then A^{-1} exist if

(a) $k = \frac{8}{5}$
(b) $k \neq 2$
(c) $k \neq \frac{-8}{5}$
(d) $k \neq -2$

10. Given that A is a square matrix of order 3x3 and $|A| = 5$ then the value of $\sum_{i=1}^{3} a_{i2} A_{i3}$, where A_{ij} denotes the cofactor of element a_{ij} is

(a) 5
(b) 0
(c) 25
(d) -5

11. The point(s) at which the function f given by $f(x) = \begin{cases} \frac{x}{|x|}, & x>0 \\ 1 & x\le 0 \end{cases}$ is continuous is/are

(a) $x = 0$
(b) $x \in R$
(c) $x \in R - \{0\}$
(d) $x = -1$ and 1

12. If $x = t^2$, $y = t^3$ then $\frac{dy}{dx} =$

(a) $\frac{3}{2}$
(b) $\frac{3}{4t}$
(c) $\frac{3}{2t}$
(d) $\frac{3t}{2}$

13. Derivative of $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$ is

(a) $\frac{\pi}{4} - x$
(b) -1
(c) $\frac{\pi}{4} + x$
(d) 1

14. The value of $f'(x)$ is $-\sqrt{3}$ at a point P on the curve $y = f(x)$, what is the angle which the tangent to the curve at P makes with positive direction of x-axis?

(a) $\frac{3\pi}{4}$
(b) $\frac{5\pi}{6}$
(c) $\frac{2\pi}{3}$
(d) $\frac{\pi}{3}$

15. A particle moving in a straight line covers a distance of x cm in t seconds where $x = t^3 + 6t^2 - 15t + 18$, what will be the velocity of the particle at the end of 2 seconds

(a) 21 cm/sec
(b) 20 cm/sec
(c) 22 cm/sec
(d) 23 cm/sec

16. If $f(x) = \cos x$, $x \in [0, 2\pi]$, the interval in which $f(x)$ is strictly decreasing is

(a) $[0\ \pi]$
(b) $(0,\ \pi)$
(c) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$
(d) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

17. The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (x^5 + x\cos x + \tan^3 x)\,dx$ is

(a) 0 (b) 2

(c) π (d) 1

18. $\int \frac{dx}{e^x + e^{-x}}$ is equal to

(a) $\tan^{-1}(e^{-x}) + c$ (b) $\tan^{-1}(e^x) + c$

(c) $\log(e^x - e^{-x}) + c$ (d) $\log(e^x + e^{-x}) + c$

Assertion-Reason Based Questions

Direction: In questions numbered 19 and 20, two statements are given one labeled Assertion (A) and the other labeled Reason(R). Select the correct answer from the following options:

(a) Both Assertion (A) and reason (R) are true and R is correct explanation of A.

(b) Both Assertion (A) and reason (R) are true but R is not correct explanation of A.

(c) Assertion (A) is true, but reason (R) is false.

(d) Assertion (A) is false, but reason (R) is true.

19. **Assertion (A):** If in an LPP ejective function attains its maximum value at two corner points of the feasible region then it attains maximum value at infinitely many pointe lying on line segment joining those two point.

Reason (R): If the value of the objective function of a LPP is same at two corner points then it is same at every point on the line joining these two points.

20. **Assertion (A):** $\int \frac{dx}{x^2+2x+3} = \log|(x+1) + \sqrt{x^2+2x+3}| + c$

Reason (R): $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

SECTION-B

21. Find the value of $\cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \tan^{-1}\left(\cot\frac{3\pi}{4}\right)$

Or

Find the domain and range of $f(x) = \sin^{-1}(1 - x^2)$.

22. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

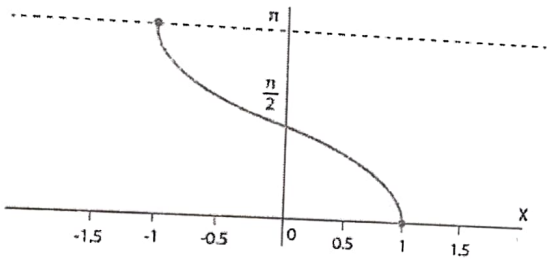
23. The side of an equilateral triangle is increasing at the rate of 3cm/s. Find the rate of change of its area when side is 20 cm.

24. Differentiate $\tan^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0 < x < 1$.

25. Evaluate: $\int \frac{e^x(1-x)^2}{(1+x^2)^2}\,dx$ Or $\int \frac{\sec^2 x}{\sqrt{4+\tan^2 x}}\,dx$

SECTION-C

26. The graph of an inverse trigonometric function $f(x)$ is given below observe the graph and answer the following questions.



(i) Which inverse trigonometric function $f(x)$ is?
What is its principal value branch?

(ii) What is the value of $f\left(\frac{-1}{2}\right)$?

(iii) If $f(x) = \frac{\pi}{4}$, then find the value of x.

27. If $x = a\sin^2\theta$, $y = a\cos^2\theta$ then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$

Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$

28. Evaluate $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$

OR

$$\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}\,dx$$

SN

29. Find the interval in which function $f(x) = 2x^3 - 9x^2 + 12x + 15$ is strictly increasing and strictly decreasing.

OR

Find the local maximum and minimum of $f(x) = \sin x + \cos x \quad x \in [0, 2\pi]$.

30. Find the value of a and b so that function f defined as :

$$f(x) = \begin{cases} \frac{x-2}{|x-2|} + a, & \text{if } x < 2 \\ a + b, & \text{if } x = 2 \\ \frac{x-2}{|x-2|} + b, & \text{if } x > 2 \end{cases}$$ is a continuous function.

31. Express matrix $A = \begin{bmatrix} 2 & 3 & -4 \\ 5 & 0 & 8 \\ 1 & 7 & 6 \end{bmatrix}$ as sum of a symmetric and a skew symmetric matrix

SECTION - D

32. Show that function $f: R \to R$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further find set A so that the given function $f : R \to A$ becomes an onto function.

OR

Let $A = \{x \in Z : 0 \le x \le 15\}$. Show that $R = \{(a,b): a,b \in A, |a - b| \text{ is a multiple of } 7\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

33. If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, find A^{-1}. Hence solve the system of equations

$$x - y = 3$$
$$2x + 3y + 4z = 17$$
$$y + 2z = 7$$

OR

Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4$

$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

34. Evaluate: $\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)} d\theta.$

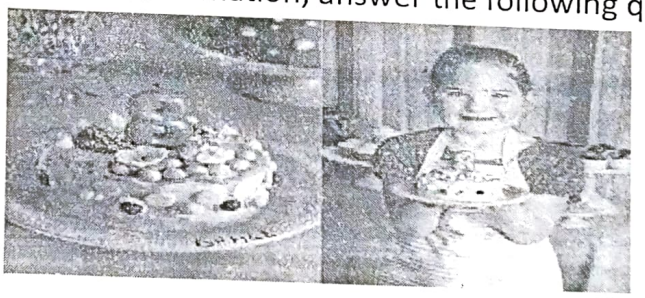
35. Solve the following Linear Programming Problem graphically:
Minimize: z = x + 2y subject to the constraints : $x + 2y \geq 100$
$2x - y \leq 0$
$2x + y \leq 200$
$x, y \geq 0$

SECTION - E

Case Study Based Questions

CASE STUDY-1

36. On her birthday, Seema decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got 10 more. However, if there were 16 children more everyone would have got 10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in₹).
Based on the above information, answer the following questions:



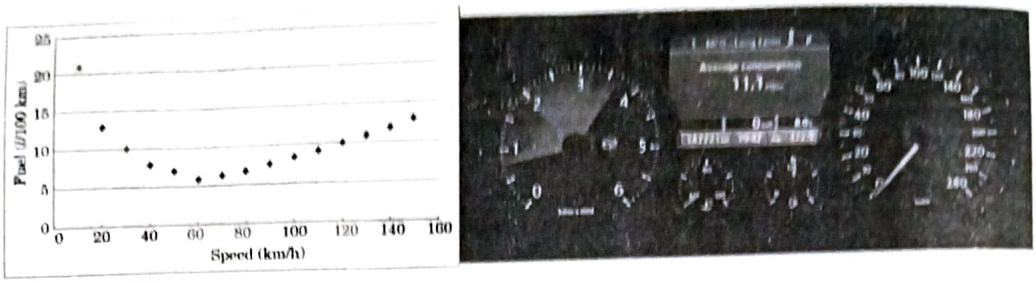
(i) Represent the above information in the form of a matrix equation.
(ii) How many children were given some money by Seema?
(iii) How much amount is given to each child by Seema ?

OR

How much amount Seema spends in distributing the money to all the students of the orphanage.

CASE STUDY - 2

37. Overspeeding increases fuel consumption and decreases fuel economy as result of tyre rolling friction and air resistance. While vehicles reach fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.



The relation between fuel consumption F (//100 km) and speed V (km/h) under someconstraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

On the basis of the above information, answer the following questions:

(i) Find F, when V = 40 km/h. (1)

(ii) Find $\frac{dF}{dV}$. (1)

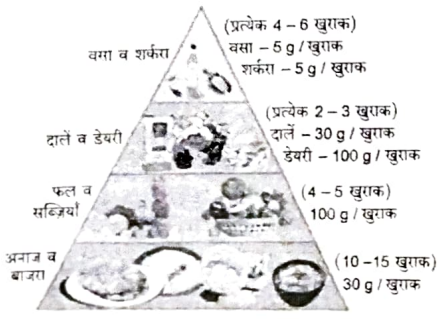
(iii) (a) Find the speed V for which fuel consumption F is minimum. (2)

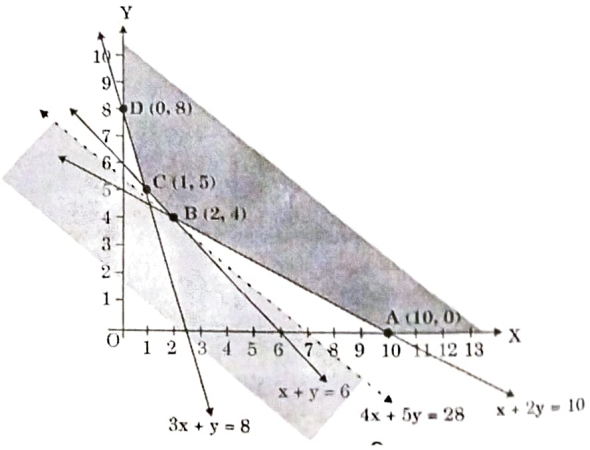
OR

(iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$. (2)

CASE STUDY-3

The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.





A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y(y kg) which are available at the rate of ₹16/kg and ₹2/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

On the basis of the above information, answer the following questions:

(i) Identify and write all the constraints which determine the given feasible region in Figure-2.

(ii) If the objective is to minimize cost Z = 16x + 20y find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region.