



श्री Venkateshwar International School

Sector-18, Dwarka, New Delhi-75

MID-TERM EXAMINATION (2025-2026)

MATHEMATICS (041)

CLASS – XII

Time: 3 Hours

Max. Marks: 80

Misbah

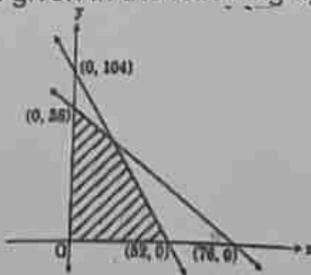
GENERAL INSTRUCTIONS:

1. Read the following instructions very carefully and strictly follow them:
2. This Question paper contains 38 questions. All questions are compulsory.
3. This Question paper is divided into five Sections - A, B, C, D and E.
4. In Section A, questions no. 1 to 18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
5. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
6. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
7. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
8. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
9. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart in 2 questions of Section E.
10. Use of calculators is not allowed.

SECTION - A

1. If for a square matrix A, $A(\text{adj } A) = \begin{bmatrix} 2026 & 0 & 0 \\ 0 & 2026 & 0 \\ 0 & 0 & 2026 \end{bmatrix}$, then the value of $|A| + |\text{adj } A|$ is
 (a) 1 (b) $2026+1$ (c) $(2026)^2 + 45$ (d) $2026+ (2026)^2$ 1
2. If $A = \{1, 3, 5\}$, then number of equivalence relations on A containing (1,3) is
 (a) 1 (b) 2 (c) 4 (d) 5 1
3. The domain of differentiability of the function $f(x) = |2x+1| \sin 3x$ is
 (a) R (b) $R - \{-\frac{1}{2}\}$ (c) $(\frac{1}{2}, \infty)$ (d) $(0, \infty)$ 1
4. If $f(x) = \begin{cases} \frac{\sin \pi x}{5x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then k is
 (a) $\frac{5}{\pi}$ (b) $\frac{\pi}{5}$ (c) 1 (d) 0 1
5. The domain of function $\cos^{-1} \sqrt{1-x}$ is
 (a) $[-1, 1]$ (b) $(-\infty, 1]$ (c) $[0, 1]$ (d) none of these 1
6. The principal value of $\sin^{-1}(\cos(\frac{43\pi}{5}))$ is
 (a) $\frac{2\pi}{5}$ (b) $-\frac{2\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $-\frac{\pi}{10}$ 1

7. If $A = \begin{bmatrix} 7 & 0 & x \\ 0 & 7 & 0 \\ 0 & 0 & y \end{bmatrix}$ is scalar matrix, then y^x is
 (a) 0 (b) 1 (c) 7 (d) ± 7 1
8. If A and B are invertible square matrices of same order, then which of the following is not correct?
 (a) $|A B^{-1}| = \frac{|A|}{|B|}$ (b) $|(AB)^{-1}| = \frac{1}{|A||B|}$ (c) $(AB)^{-1} = B^{-1} A^{-1}$ (d) $(A+B)^{-1} = B^{-1} + A^{-1}$ 1
9. The sum of two skew-symmetric matrices of same order is always a/an
 (a) skew-symmetric matrix (b) symmetric matrix
 (c) null matrix (d) identity matrix 1
10. The value of $\int_0^{\frac{\pi}{2}} \sqrt{1 - \sin 2x}$ is
 (a) $2\sqrt{2}$ (b) $2(\sqrt{2} + 1)$ (c) 2 (d) $2(\sqrt{2} - 1)$ 1
11. The value of $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$ is
 (a) $\sin\sqrt{x} + c$ (b) $-\sin\sqrt{x} + c$ (c) $2\cos\sqrt{x} + c$ (d) $2\sin\sqrt{x} + c$ 1
12. The value of $\int_0^2 |x - 2| dx$ is
 (a) 1 (b) 2 (c) -1 (d) -2 1
13. If a function $f: R \rightarrow R$ is defined as $f(x) = x^3 + 1$. Then the function has
 (a) No minimum value
 (b) No maximum value
 (c) Both maximum and minimum values
 (d) Neither maximum value nor minimum value 1
14. The value of p so that the function $f(x) = \sin x - \cos x - px + q$ decreases for all real values of x is
 (a) $p \geq \sqrt{2}$ (b) $p < \sqrt{2}$ (c) $p \geq 1$ (d) $p < 1$ 1
15. The feasible region of an LPP is given in the following figure:



Then, the constraints of the LPP are $x \geq 0, y \geq 0$ and

- (a) $2x + y \leq 52$ and $x + 2y \leq 76$ (b) $2x + y \leq 104$ and $x + 2y \leq 76$
 (c) $x + 2y \leq 104$ and $2x + y \leq 76$ (d) $x + 2y \leq 104$ and $2x + y \leq 38$ 1
16. A linear programming problem is as follows:
 Minimise $Z = 30x + 50y$, subject to the constraints,
 $3x + 5y \geq 15, 2x + 3y \leq 18, x \geq 0, y \geq 0$
 in the feasible region, the minimum value of Z occurs at
 (a) A unique point (b) no point
 (c) Infinitely many points (d) two points only 1

17. If for real numbers x, y define xRy iff $x-y+\sqrt{2}$ is an irrational number. Then the relation R is
 (a) reflexive and symmetric (b) symmetric and not transitive
 (c) transitive and reflexive (d) not symmetric and not transitive 1
18. The function $f: R \rightarrow R$ defined by $f(x) = 4+3 \cos x$ is
 (a) bijective (b) one-one but not onto
 (c) onto but not one-one (d) neither one-one nor onto 1

Assertion reasoning Based questions

Statement I is called Assertion (A) and **Statement II** is called Reason (R). Read the given statement carefully and choose the correct answer from the four options given below.

- (a) Both the statements are true and Statement II is the correct explanation of Statement I.
 (b) Both the statements are true and Statement II is not the correct explanation of Statement I.
 (c) Statement I is true, Statement II is false.
 (d) Statement II is true; Statement I is false.

19. **Assertion (A):** Minimum value of function $f(x) = |\sin x|, x \in R$ is -1.
Reason (R): Rate of change of y with respect to x is $\frac{dy}{dx}$. 1
20. **Assertion (A):** If A is a square matrix of order 3×3 and 2 is any scalar then the value of $|2A| = 8|A|$.
Reason (R): If k is a scalar and A is a square matrix of order $n \times n$. Then $|kA| = k^{n-1}|A|$. 1

Section- B

(This section comprises 5 very short answer (VSA) type questions of 2 marks each.)

21. Let $f: R \rightarrow R$ be a function defined by $f(x) = \frac{x}{x+1}$. Check its injectivity and surjectivity. 2
22. Find the derivative of $\sin^2 x$ w.r.t. $e^{\cos x}$. 2
 OR
 Differentiate $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ w.r.t. $x, x > 0$.
23. If $\frac{d}{dx} F(x) = \frac{1}{\sqrt{2x-x^2}}$ and $F(1) = 0$, find $F(x)$. 2
24. Show that the elements along the main diagonal of a skew symmetric matrix are all zero. 2
 OR
 If $f(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then prove that $f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$.
25. If the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \text{adj } A$ and $C = 3A$, then find the value of $\frac{|adj B|}{|C|}$. 2

Section - C

(This section comprises 6 short answer (SA) type questions of 3 marks each.)

26. Let \mathbb{R} be the set of all real numbers and R be the relation on \mathbb{R} defined as $R = \{(a, b) : |a| \leq b\}$. Check whether R is reflexive, symmetric and transitive relation. 3
27. If $x \cos(p + y) + \cos p \sin(p + y) = 0$, then prove that $\cos p \frac{dy}{dx} = -\cos^2(p + y)$ where p is a constant. 3
- OR
- If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$.
28. Evaluate: $\int \frac{x^4}{(x-1)(x^2+1)} dx$ 3
- OR
- Evaluate: $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 x + 4 \sin^2 x}$
29. Find the intervals in which the function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ is
 (A) Strictly increasing (B) Strictly decreasing 3
30. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ then show that $A^2 - 4A + 7I = 0$. Using this result, calculate A^4 . 3
31. Solve the following Linear Programming Problem (LPP) by graphical method:
 Minimise $Z = 2x + y$, subject to the constraints
 $6x + 2y \geq 18$, $3x + 3y \geq 21$, $2x + 4y \geq 16$ and $x \geq 0$ and $y \geq 0$ 3

Section- D

(This section comprises 4 long answer (LA) type questions of 5 marks each.)

32. (A) Evaluate: $\int e^{\cot^{-1} x} \left(\frac{1-x+x^2}{1+x^2} \right) dx$. 2.5
 (B) Evaluate: $\int_0^{\frac{\pi}{2}} \left(\frac{5 \sin x + 3 \cos x}{\sin x + \cos x} \right) dx$. 2.5
- OR
- (A) Evaluate: $\int_0^{\pi/4} \frac{1}{\cos^3 x \sqrt{2 \sin 2x}} dx$.
 (B) Evaluate: $\int \frac{x + \sin x}{1 + \cos x} dx$.
33. If the sum of the lengths of hypotenuse and a side of a right angle triangle is given, show that area of triangle is maximum, when the angle between them is $\frac{\pi}{3}$. 5
- OR
- Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its side. Also, find the maximum volume.

34. (A) Find the values of p and q for which $f(x)$ is continuous at $x = \frac{\pi}{2}$. 3
- $$f(x) = \begin{cases} \frac{1 - \sin^3 x}{3 \cos^2 x} & \text{if } x < \frac{\pi}{2} \\ p & \text{if } x = \frac{\pi}{2} \\ \frac{q(1 - \sin x)}{(\pi - 2x)^2} & \text{if } x > \frac{\pi}{2} \end{cases}$$

(B) If $e^y(x+1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$. 2

35. (A) Simplify: $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x \neq 0$ 2.5

(B) If A is a symmetric matrix and B is a skew-symmetric matrix such that $A+B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then find $|AB|$. 2.5

Section- E

(This section comprises 3 case study-based questions of 4 marks each.)

36. In a residential society comprising of 100 houses, there were 60 children between the ages of 10-15 years. They were inspired by their teachers to start composting to ensure that biodegradable waste is recycled. For this purpose, instead of each child doing it for only his/her house, children convinced the Residents' Welfare Association to do it as a society initiative. For this they identified a square area in the local park. Local authorities charged amount of ₹ 50 per square meter for space so that there is no misuse of the space and Residents' Welfare Association takes it seriously. Association hired a labourer for digging out 250 m^3 and he charged ₹ $400 \times (\text{depth})^2$. Association will like to have minimum cost. Based on the above information, answer the following questions:

(A) Let side of square plot is x meter and its depth is h meter, then find the cost c for the pit. 1

(B) Find the value of h for which $\frac{dc}{dh} = 0$. 1

(C) What value of x be required for minimum cost function? 2

OR

(C) What total minimum cost will be given for digging the pit (in ₹)?

37. In a school project, Mehak sketched the graph for $\tan^{-1}(x)$. She sketches the graph by considering bijectivity of the function in some particular domain.

Based on the above information, answer the following questions:

(A) Express: $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$, $-\frac{3\pi}{2} \leq x < \frac{\pi}{2}$ in simplest form. 2

(B) Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2} \cos^{-1}\left(\frac{1-x}{1+x}\right)$, $x \in [0,1]$. 2

38. In schools' students not only get academic guidance but also, they get to know about carrier options and right goals as per their interest and academic record. A school conducted classes in two sections A and B and the fees for different type of combinations of subjects are different. There are 20 type 1 combination and 5 for type 2 combination of students and total monthly collection is ₹ 9,000, where as in section B, there are 5 type 1 combination and 25 for type 2 combination of students and monthly collection is ₹ 26,000.

Based on the above information, answer the following questions:

(A) If ₹ x and ₹ y be the fees for type 1 combination and type 2 combination of students respectively, then express the information provided in problem in system of linear equation. 1

(B) Express the system of linear equations obtained in (A) as matrix equation. 1

(C) Find A^{-1} . 2

OR

(C) Find the fee for type 1 combination and for type 2 combination of students.